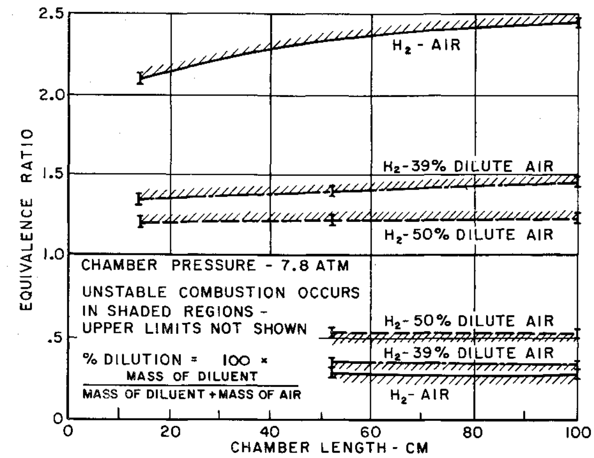

Fig. 1 Longitudinal mode instability regions for hydrogen-dilute air mixtures.

in the interpretation of instability data obtained from gas rockets.

Recently the difference has been resolved in terms of a model in which the driving mechanism for instability is related to chemical kinetic factors.[4] In this model, the longitudinal stability limits are defined in terms of a critical value for a rate parameter E/RT (E = an over-all activation energy for the propellant combination, T = combustion temperature, and R = gas constant). For a given propellant combination E may be assumed constant, and, hence, the stability limits are defined in terms of a critical combustion temperature, T_{crit}. For $T < T_{crit.}$, the combustion is unstable, and for $T > T_{crit.}$, the combustion is stable.[4] It is concluded, therefore, that there will be two unstable regions, one on either side of stoichiometric, whenever the maximum combustion temperature for the propellant combination is greater than T_{crit}. When the maximum combustion temperature is less than T_{crit}, one unstable region, located around stoichiometric, will be observed. For propellant combinations for which two unstable regions are observed, it is possible to collapse the two regions to a single region through the addition of an inert diluent. The diluent reduces the over-all range of combustion temperatures, and thus causes the stability limits to shift toward stoichiometric. In the experiments of Fig. 1, increasing amounts of nitrogen were added to hydrogen-air, and, as predicted, the stability limits approach one another until finally the two regions coalesce. The combustion temperature at the stability limits is found experimentally to be approximately 1400°K for all degrees of dilution. For the case when the two unstable regions coalesce (60% dilute air), the maximum combustion temperature is somewhat less than 1400°K. Figure 2 shows similar results for methane-dilute oxygen mixtures. For methane-($0.4O_2 + 0.6N_2$) two

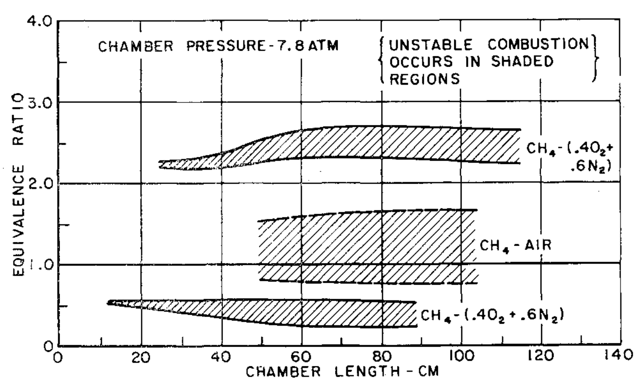

Fig. 2 Longitudinal mode instability regions for methane-dilute oxygen mixtures.

unstable regions are found, with a combustion temperature at the stability limits of approximately 2200°K. For methane-air one unstable region is observed, and a maximum combustion temperature somewhat less than 2200°K is measured. It is probable that one unstable region is characteristic of the combustion of gaseous paraffins and air, since the over-all activation energies and maximum combustion temperatures are approximately the same. One unstable region has been observed by Zucrow and Osborn[2] for methane-air, ethane-air, and propane-air.

In conclusion, it is seen that the observations of the various investigators with respect to the number and location of unstable regions are consistent, and that the observations can be explained in terms of an instability model in which the driving mechanism depends on chemical kinetic factors.

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Comment on "Proposal Concerning Laminar Wakes behind Bluff Bodies at Large Reynolds Numbers"

PETER D. RICHARDSON*
Brown University, Providence, R. I.

THERE has been considerable interest in the determination of flow patterns at large Reynolds numbers, when the motion is completely steady. Under these circumstances, flows are required to satisfy the Navier-Stokes equation without regard to questions of the stability of the flow; features characteristic of turbulent motion are excluded from discussion. One configuration of general interest is that of a bluff body held in an infinite, steady stream. Batchelor[1] reviewed this problem and gave strong reasons for believing that the region behind the body is one where the streamlines are closed. This feature is observed in solutions and in visualization at low Reynolds numbers. Batchelor also discussed the form that this closed region, or bubble, would take as the Reynolds number tends to infinity. He concluded that the bubble remains finite in length. An essential part of his argument in support of this model was that the bubble contains an extensive region in which viscous forces are small (in the boundary-layer sense). This work of Batchelor was published shortly after a finite-difference solution of a related wake problem by Allen and Southwell[2] for Reynolds numbers of 10, 100, and 1000. More recently, Acrivos et al.[3,4] have published measurements for wakes behind two-dimensional bluff obstacles and have suggested that the steady laminar wake has a length increasing linearly with Reynolds number. The data on which the latter suggestion is based include measurements taken with a stabilizing plate in the wake region, and the steady wake flow in its presence cannot be identical with that in its absence, so that it may have influenced the variation of length with Reynolds number. The purpose of this note is to outline an alternative proposal, which has its

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* Assistant Professor of Engineering.

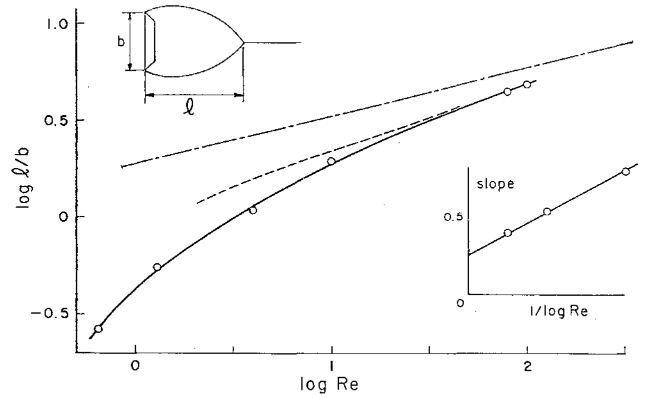

Fig. 1 The behavior of a steady laminar wake behind a bluff flat plate, from Tietjens. The inset graph shows the slope of the solid line as a function of reciprocal log *Re*; the straight chain line corresponds to Eq. (1) and the dashed line to its modification.

roots in other experimental observations and, in a sense, lies between the proposals mentioned. This alternative proposal was first noted in 1961.[5]

The surface of any separation bubble is subjected to a pressure distribution that is essentially determined by the bubble shape. Close to the point of separation, the pressure gradient is slightly negative or zero, and the pressure is less than far upstream; in the region where the streamlines of the external inviscid flow come towards the center plane of the flow the pressure gradient is positive, becoming zero in the vicinity of the closure point of the bubble. Provided that the displacement thickness of the post-bubble wake is always less than the greatest thickness of the bubble, an extensive portion of the bubble is subjected to a positive pressure gradient. It is well known that for attached laminar boundary layers with constant positive pressure gradients there is no unique solution,[6] and Stewartson[7] showed that part of the set of solutions includes profiles with reversed flow in the vicinity of the surface. Kennedy[8] tabulated some (unique) similarity solutions of wake-like form for positive pressure gradients. Correspondingly, it appears possible for the rear portion of the bubble to contain a reversed flow in its center region, being driven forwards by the pressure gradient. It is only when the pressure gradient is extremely small that there is a significant portion of the bubble cross section in which the velocity gradient is small. At indefinitely large Reynolds number the bubble assumes the shape of a slender rapier.

At the present time, there is insufficient quantitative information available about the influence of varying the (dimensionless) pressure gradient along the stream on the flow for a solution to be presented. However, it would be useful to know the rate at which the bubble length increases at large Reynolds number. Heuristically, this might be inferred from measurements of flow at lower Reynolds number. One of the most extensive and consistent descriptions of steady flow behind a bluff body has been provided by Tietjens.[9] A graph representing the dimensionless bubble length as a function of Reynolds number for flow behind a bluff flat plate is shown in Fig. 1. Over a very small interval of Reynolds number this function can be represented as $l/b \propto Re^m$, where m is a function of the Reynolds number. The inset on Fig. 1 shows m as a function of the reciprocal Reynolds number. From these graphs it is apparent that at high Reynolds number m tends asymptotically downwards to a value of about $\frac{1}{4}$, and that the bubble length tends to

$$l/b \simeq 1.8Re^{1/4} \tag{1}$$

At lower Reynolds numbers, deviations from this may be expected because 1) the centerline length of the bubble is significantly smaller than the separating streamline length, and 2) the assumption (appropriate to boundary-layer analysis) that velocity gradients in one direction are dominant is no longer valid. If it is assumed as an approximation that the effective (separating streamline) length is equal to the centerline length l plus the body width b, the modified asymptotic behavior is given by the dashed line of Fig. 1; this is sufficient to account for most of the deviation at the larger Reynolds numbers.

It may be noted that the growth rate given by Eq. (1) lies between the suggestions of Batchelor[1] and Acrivos et al.[4] It does not appear inconsistent with the proposed physical bubble behavior or with any available computations.

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Wedge and Cone Theory for $M_\infty = \infty$

Joseph W. Cleary*
NASA Ames Research Center, Moffett Field, Calif.

Nomenclature

C_p = surface pressure coefficient, $p - p_\infty/q_\infty$
M = Mach number
p = pressure
q = dynamic pressure
u, v = velocity components parallel and perpendicular to freestream
V = velocity
γ = ratio of specific heats
δ = cone or wedge angle
θ = shock angle

Subscripts

∞ = freestream conditions
c = cone
D = detachment
e = effective value
w = wedge
2 = conditions immediately behind oblique shock

The tabulated values of the ratio of cone surface pressure to pressure at the shock p_c/p_2 presented in Ref. 1 for γ = 1.405 and 1.333 at $M_\infty = \infty$ indicate that this ratio is sensibly

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* Research Scientist. Member AIAA.